Chapter 6
The Economics of Property Law: Property Rights and Incentives

These notes describe a general class of contracts (or leases) between a landowner and a tenant, both of whom contribute inputs into the productive use of the land. The prototypical example is a landlord and a tenant farmer. Let

$V(x,y)$ = gross value of the land;
x = cost of farmer's inputs;
y = cost of landlord's inputs;

where

$V_x>0,\ V_{xx}<0,\ V_y>0,\ V_{yy}<0$

Thus, inputs from both parties increase output, but at a decreasing rate.

The socially optimal choices of x and y maximize the net value of output:

$$V(x,y) - x - y \qquad (6.1)$$

The first-order condition for x and y are

$$V_x - 1 = 0 \qquad (6.2)$$

$$V_y - 1 = 0. \qquad (6.3)$$

Solved simultaneously, these determine x^* and y^*.

Contracts

Consider a generalized share contract that allocated a fraction s of gross output to the landlord, and the remaining share, $1-s$, to the tenant. Also, let R be a transfer payment (rent) paid from the tenant to the landlord. The expected returns to the landlord and the tenant, respectively, are therefore

$$U_L = sV(x,y) - y + R \qquad (6.4)$$

$$U_T = (1-s)V(x,y) - x - R \qquad (6.5)$$

The incentive compatibility constraints that determine their input choices are as follows:

$$sV_y - 1 = 0 \qquad \text{(landlord)} \qquad (6.6)$$

$$(1-s)V_x - 1 = 0 \qquad \text{(tenant)} \qquad (6.7)$$

These determine $\hat{y}(s)$ and $\hat{x}(s)$, respectively. It follows that $\frac{\partial \hat{y}}{\partial s} > 0$ and $\frac{\partial \hat{x}}{\partial s} < 0$. Finally, if we treat the landlord as the principal, the participation constraint for the tenant is $U_T \geq \bar{U}_T$.

We can now write the landlord's problem as choosing s and R to maximize U_L subject to the incentive compatibility constraints in (6.6) and (6.7), and the tenant's participation constraint. The Lagrangian function for this problem is

$$L = sV(\hat{x}, \hat{y}) - \hat{y} + R + \lambda[(1-s)V(\hat{x}, \hat{y}) - \hat{x} - R - \bar{U}_T] \tag{6.8}$$

The first-order condition for R is

$$1 - \lambda = 0 \tag{6.9}$$

which implies that λ=1. Thus, the tenant's participation constraint is binding. It follows that the rent is given by

$$R = (1-s)V(\hat{x}, \hat{y}) - \hat{x} - \bar{U}_T \tag{6.10}$$

The first-order condition for s (after substituting λ=1) is given by

$$(1-s)V_y\left(\frac{\partial \hat{y}}{\partial s}\right) + sV_x\left(\frac{\partial \hat{x}}{\partial s}\right) = 0 \tag{6.11}$$

In the most general case where V_y and V_x are both positive (i.e., inputs from both the landlord and tenant contribute to value), the optimal contract involves $0<s<1$; that is, a shared contract. From (6.10), R may be positive or negative. In the case where R=0, this results in a *pure sharecropping contract*. Note that under this type of contract, both parties invest some effort, though both underinvest relative to the efficient levels.

Another important special case arises when V_y=0 and V_x>0. That is, only the tenant's input matters. This would be the case for most agricultural contracts, where the tenant farmer plants, tends, and harvests the crops. In that case, (6.11) reduces to

$$sV_x\left(\frac{\partial \hat{x}}{\partial s}\right) = 0 \tag{6.12}$$

which implies that s=0. From (6.10) we have

$$R = V(\hat{x}, \hat{y}) - \hat{x} - \bar{U}_T \tag{6.13}$$

which is certainly positive (for otherwise there would be no net gains from the contract). This represents a *pure rental contract*, under which the tenant pays a fixed rent and then retains the entire net harvest. In other words, the tenant is the residual claimant. In this case, the tenant's input is efficient—that is, (6.7) coincides with (6.3).